

Opener

Non-Calculator

A particle moves along the x -axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

$$v(t) = 2t - 6 = 0$$

$$2t = 6$$

$$t = 3$$

Calculator

A particle moves along a line so that at time t , where $0 \leq t \leq \pi$, its position is given by $s(t) = -4 \cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

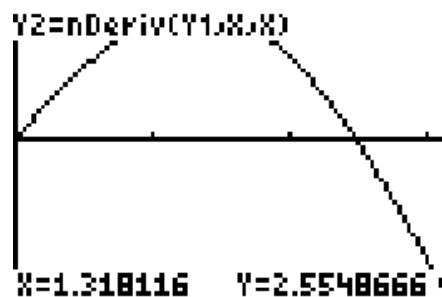
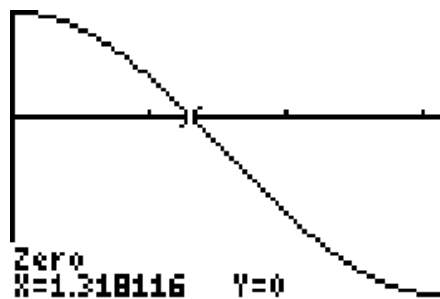
(A) -5.19

(B) 0.74

(C) 1.32

(D) 2.55

(E) 8.13



3-5 Derivatives of the Trig Functions

Learning Objectives:

I can calculate the derivatives of the trig functions.

I can write the equation of the normal line to a curve.

Derivatives of the Trig Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

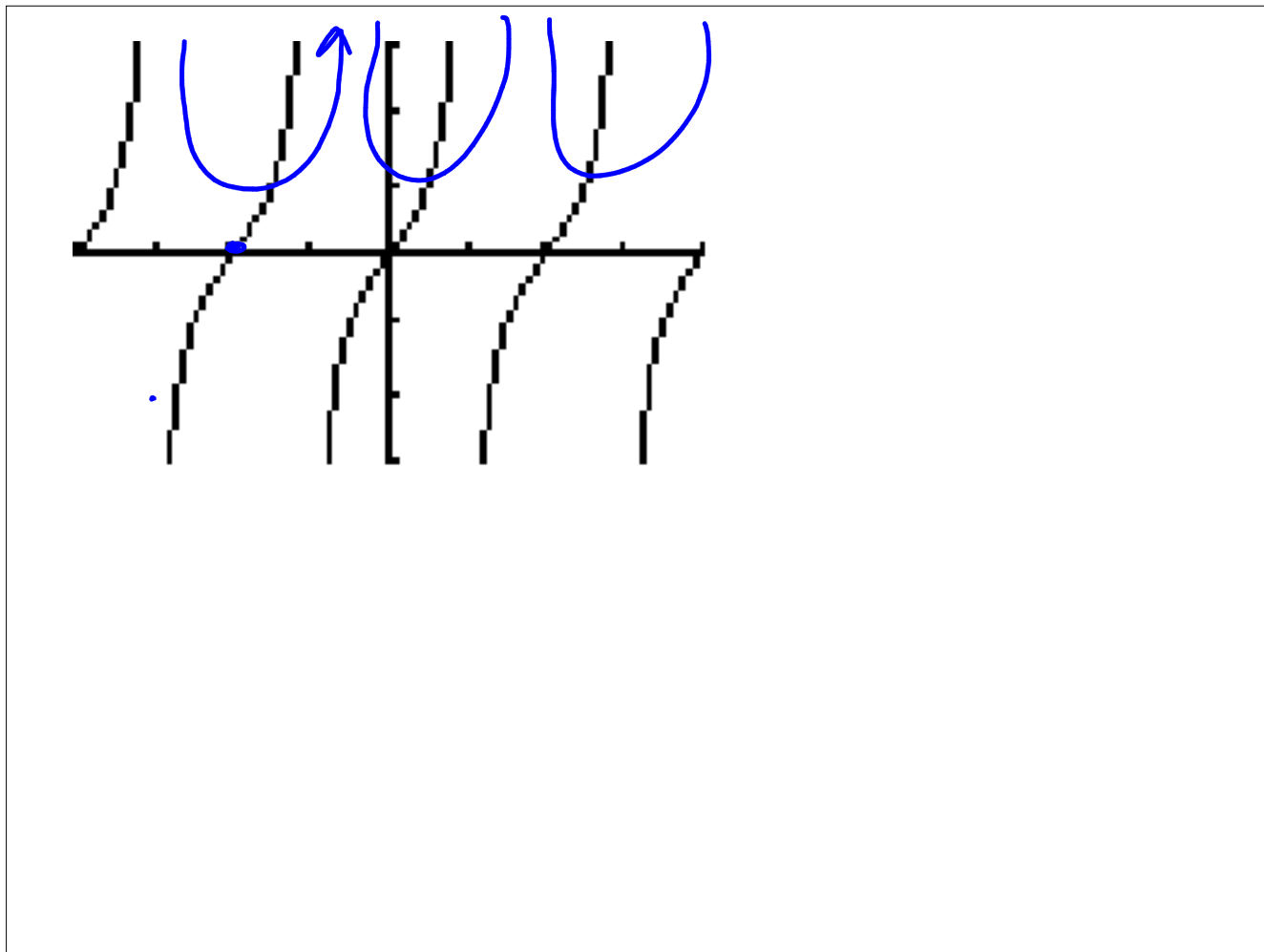
$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$$



Ex1. Find the derivative

$$1.) f(x) = \cos x + \sin x - \tan x + x^2$$
$$f' = -\sin x + \cos x - \sec^2 x + 2x$$

$$2.) y = x^2 \cos x$$

f g

$$y' = f' \cdot g + f \cdot g'$$

$$f = x^2$$
$$f' = 2x$$

$$g = \cos x$$
$$g' = -\sin x$$

$$y' = 2x \cos x + x^2 \cdot -\sin x$$
$$y' = 2x \cos x - x^2 \sin x$$

$$3.) y = \frac{x^3}{\sin x}$$

f g

$$f = x^3$$
$$f' = 3x^2$$

$$y' = \frac{f'g - fg'}{g^2}$$

$$g = \sin x$$
$$g' = \cos x$$

$$y' = \frac{3x^2 \sin x - x^3 \cos x}{\sin^2 x}$$

Develop the rule for the derivative $y = \tan x$ using quotient rule

$$5.) y = \tan x = \frac{\overset{f}{\sin x}}{\underset{g}{\cos x}}$$

$$y' = \frac{f'g - fg'}{g^2}$$

$$f = \sin x$$

$$f' = \cos x$$

$$g = \cos x$$

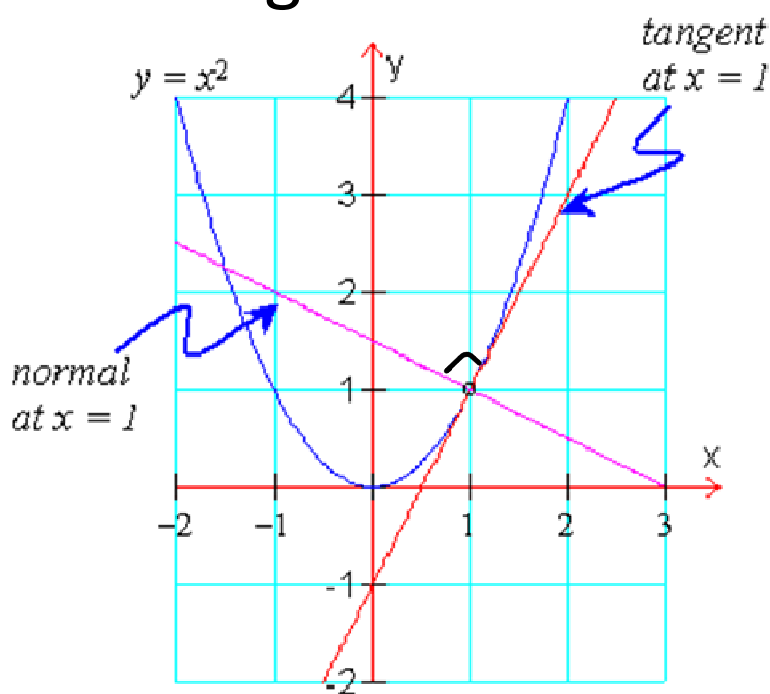
$$g' = -\sin x$$

$$y' = \frac{\cos x \cdot \cos x - \sin x \cdot -\sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

Tangent Line vs Normal Line



Ex2a.) Find the equation of the tangent line to the curve $y=x^2\sin x$ at $x=\frac{\pi}{3}$

b.) Find the equation of the normal line to the curve $y=x^2\sin x$ at $x=\frac{\pi}{3}$

(No Graphing Calculator)

tangent line to $y=x^2\sin x$ at $x=\frac{\pi}{3}$. (No G.C.)

$$y' = f'g + fg'$$

$$= 2x\sin x + x^2\cos x$$

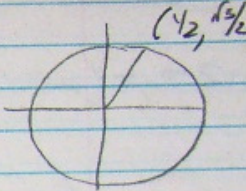
$$y' = 2\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right) + \left(\frac{\pi}{3}\right)^2\cos\left(\frac{\pi}{3}\right)$$

$$y = \frac{2\pi\sqrt{3}}{3} + \frac{\pi^2}{9} \cdot \frac{1}{2}$$

$$y' = \frac{\pi\sqrt{3}}{3} + \frac{\pi^2}{18}$$

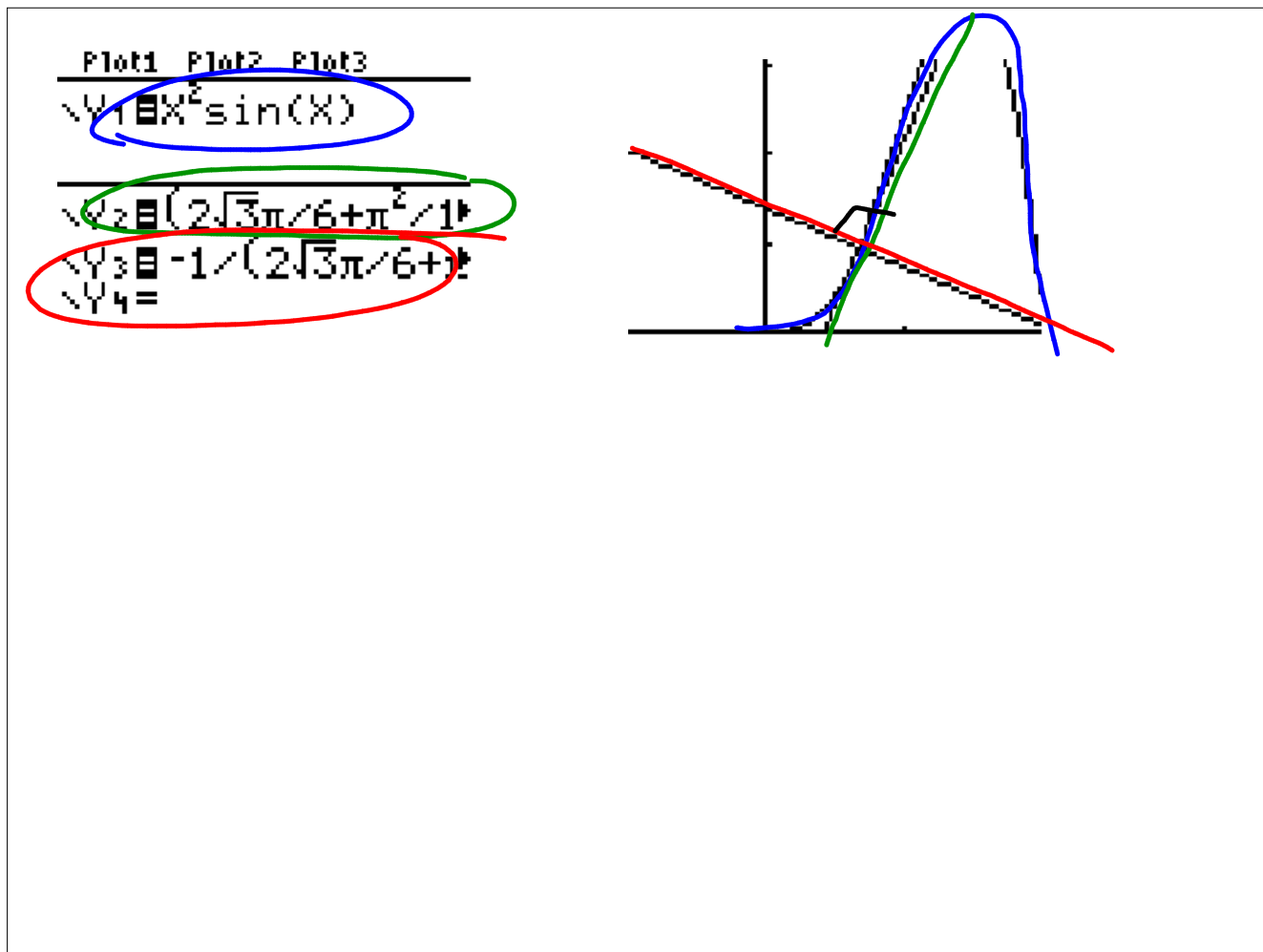
$y\left(\frac{\pi}{3}\right) = \left(\frac{\pi}{3}\right)^2\sin\left(\frac{\pi}{3}\right)$
 $\frac{\pi^2}{9} \cdot \frac{\sqrt{3}}{2} = \frac{\pi^2\sqrt{3}}{18}$

$y - y_1 = m(x - x_1)$

$$y - \frac{\pi^2\sqrt{3}}{18} = \left(\frac{\pi\sqrt{3}}{3} + \frac{\pi^2}{18}\right)\left(x - \frac{\pi}{3}\right)$$


$$y = \frac{\pi^2\sqrt{3}}{18} = \frac{-1}{\frac{\pi\sqrt{3}}{3} + \frac{\pi^2}{18}} \left(x - \frac{\pi}{3}\right)$$

normal line



Homework

pg 146 # 1-11, 21, 23, 27, 38, 46-49